Math 131 notes

Jason Riedy

20 August, 2008

Contents

Notes also available as PDF.

1 Review: Inductive and deductive reasoning

Inductive making an "educated" guess from prior observations.

Deductive if premises are satisfied, conclusion follows.

Mathematics is a combination of both forms of reasoning in no particular order.

- Problems to find: inductive
- Problems to prove: deductive
- Finding a proof... both!

Recall examples:

- Example of inductive reasoning: Extending a sequence from examples.
- Example of deductive reasoning: Deriving a rule for computing a sequence.

Always take care with your premises. Be sure you understand the framework before exploring with guesses.

2 Inductive reasoning on sequences

Purpose: Define some terminology. See how different sequences grow.

Sequence list of numbers

Term one of the numbers in a list

Examples:

- 3, 5, 7, 9, 11, ...
- 4, 12, 36, 108, ...

(Elipsis is **three** dots and is **not** followed by a comma. Text's use is incorrect on page 10.)

Two common types of sequences:

Arithmetic Defined by an initial number and a constant increment.

Geometric Defined by an initial number and a constant multiple.

In our examples:

- 3, 5, 7, 9, 11, ... : Arithmetic, starts with 3, incremented by 2.
- 4, 12, 36, 108, ... : Geometric, starts with 4, multiplied by 3.

On growth:

- Note how the arithmetic sequence's growth is "smooth", linear.
- The geometric sequence grows much more quickly, exponential.

3 A tool for sequences: successive differences

Technique is useful for finding an arithmetic sequence buried in a more complicated appearing sequence of numbers.

This is an example of reducing to a known, simpler problem. We will explore this and other general problem solving methods shortly.

Simple example with an arithmetic sequence:

3	
5	2
7	2
9	2
11	2

Note that the last column provides the increment.

Another example, not directly arithmetic:

2			
6	4		
22	16	12	
56	34	18	6
114	58	24	6

The third column is an arithmetic sequence.

To obtain the next term, fill in the table from the right:

2			
6	4		
22	16	12	
56	34	18	6
114	58	24	6
202	**88**	**30**	**6**

4 Successive differences are not useful for everything.

What if we apply this to the geometric sequence above?

4			*Completing the*
12	8		*table is*
36	24	16	*not necessary.*
108	72	48	*Look at the*
324	216	144	*growth*
972	648	432	

- Note that each successive column grows just as quickly as the first.
- Divide the first column by 4, second by 8, *etc.*, and what happens? The columns are the same.
- Successive differences of a geometric sequence still are geometric sequences.

5 An application where successive differences work, amazingly.

- Will return to the "number patterns" examples in the future.
- Skipping to the "figurate numbers" as another example of successive differences.
- Also to define common terminology.

For the terminology, consider the following table header from the context of sequences:

n	T_n	S_n

- In general, n in mathematics is an integer that counts something.
- Here, the term (individual number) within a sequence (list of numbers).
- $n = 1$ is the first term, $n = 2$ the second, *etc.*
- A sequence often is named with a letter. Here T and S for triangular and square. Will explain the names in a moment.
- A particular term n in sequence T is T_n.

To explain the names, start with two points. Draw triangles off of one, squares off the other. Fill in the following table:

n	T_n	S_n
1	1	1
2	3	4
3	6	9
4	10	16
5	15	25

The text provides formula. Plug in n, get a number. Or apply successive differences:

n	S_n	$\Delta_n^{(1)} = S_n - S_{n-1}$	$\Delta^{(2)} = \Delta_n^{(1)} - \Delta_{n-1}^{(1)}$
1	1		
2	4	3	
3	9	5	2
4	16	7	2
5	25	9	2

Terminology notes: A superscript with parenthesis often indicates a step in a process. Here it's the depth of the difference. And Δ (Greek D, "delta") is a traditional letter for differences.

6 Next time: Problem solving techniques.

7 Homework

Practice is absolutely critical in this class.

Groups are fine, turn in your own work. Homework is due in or before class on Mondays.

- Exercises for Section 1.1:
  - Even problems 2-12. One short sentence of your own declaring *why* you decide the reasoning is inductive or deductive. Feel free to scoff where appropriate.
- Explain why the Section 1.1's example of "2, 9, 16, 23, 30" is a trick question.
- Exercises for Section 1.2:
  - Problems 2, 9, and 10.
  - Problems 14 and 16.
  - Problems 29 (appropriate formula is above problem 21), and 30.
  - Problems 32, 39, and 51.

Note that you *may* email homework. However, I don't use Microsoft™ products (*e.g.* Word), and software packages are notoriously finicky about translating mathematics.

If you're typing it (which I advise just for practice in whatever tools you use), you likely want to turn in a printout. If you do want to email your submission, please produce a PDF or PostScript document.