Math 131 homework: Irrationals, decimals, and floating-point

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1 Homework

Notes also available as PDF.

Practice is absolutely critical in this class.

Groups are fine, turn in your own work. Homework is due in or before class on Mondays.

- Problem set 6.4:
  - 15, 16
  - 25-28 (Note: If you write these as $a^{\frac{1}{2}}$, you can use the rule $(ab)^k = a^k b^k$ and the factorization of each number to simplify the expressions. That summarizes the text's examples.)
  - 41-44 (Again, think of these as fractional exponents. Use the factorizations of each number under the square root, and then use the distributive property to pull out common factors.)
  - 49
  - 81

- Problem set 6.3:
  - 75, 76, 79, 80 (calculators are fine, but correctly denote what repeats)
  - 86, 87, 88
  - 95, 96

- Problem set 6.5:
  - 1-5

- On rounding and floating point arithmetic:
  - Round each of the following to the nearest tenth (one place after the decimal) using **round to nearest even**, **round to zero (truncation)**, and **round half-up**:
    * 86.548
    * 86.554
    * 86.55
  - Compute the following quantities with a computer or a calculator. **Write what type of computer/calculator you used and the software package if it's a computer.** Compute it as shown. Do not simplify the expression before computing it, and do not re-enter the intermediate results into the calculator or computer program. Also compute the expressions that do not include 10^{16} by hand exactly. There should be a difference between the exact result and the displayed result in some of these cases. Remember to work from the innermost parentheses outward.
    * $(\overbrace{0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1}^{10 \text{ times}}) - 1$
    * $((\overbrace{0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1}^{10 \text{ times}}) - 1) \times 10^{16}$, where 10^{16} often is entered as 1e16. If the result overflows (signals an error) on various calculators, replace 10^{16} by 10^8 in this and later portions.
    * $(((2 \div 3) - 1) \times 3) + 1$

* $\left(\left(\left((2 \div 3) - 1\right) \times 3\right) + 1\right) \times 10^{16}$

* $\left(\left((6 \div 7) - 1\right) \times 7\right) + 1$

* $\left(\left(\left((6 \div 7) - 1\right) \times 7\right) + 1\right) \times 10^{16}$

The object of this first part is to demonstrate round-off error. The first to problems, adding 0.1 repeatedly, may see no error if the device calculates in decimal. The latter four parts should see some error regardless of the base used.

– Now copy down the number displayed by the first calculation in each of the following. Re-enter it as x in the second calculation.

* $1 \div 3$, then $1 \div 3 - x$ where x is the number displayed.

* If you have a calculator or program with π, π, then $\pi - x$ where x is the number displayed.

The object here is to see that the number displayed often is not the number the computer or calculator has stored.

Note that you *may* email homework. However, I don't use Microsoft$^{\text{TM}}$ products (*e.g.* Word), and software packages are notoriously finicky about translating mathematics.

If you're typing it (which I advise just for practice in whatever tools you use), you likely want to turn in a printout. If you do want to email your submission, please produce a PDF or PostScript document.