Solutions for the tenth week's homework
Math 131

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Also available as PDF.

1 Exercises 6.4

Problem 15 The sum is $0.\overline{8}$, which is rational. So the sum of two irrationals may be rational. That should not be surprising; $2-\sqrt{2}$ and $\sqrt{2}$ are irrational, but their sum is the rational 2.

Problem 16 The sum is $0.262662666\ldots$, which is not repeating or terminating and thus is irrational. So the sum of two irrationals, like $\sqrt{2}$ and $\sqrt{2}$, can be irrational, like $2\sqrt{2}$.

Problem 25 $\sqrt{50} = \sqrt{2\cdot 5^2} = (2\cdot 5^2)^{\frac{1}{2}} = 5\sqrt{2} \approx 7.07$.

Problem 26 $\sqrt{32} = \sqrt{2^5} = 2^2\sqrt{2} \approx 5.66$

Problem 27 $\sqrt{75} = \sqrt{3\cdot 5^2} = 5\sqrt{3} \approx 8.66$

Problem 28 $\sqrt{150} = \sqrt{2\cdot 3\cdot 5^2} = 5\sqrt{6} \approx 12.25$.

Problem 41 $3\sqrt{18}+\sqrt{2} = 3\cdot(2\cdot 3^2)^{\frac{1}{2}} + 2^{\frac{1}{2}} = 9\cdot 2^{\frac{1}{2}} + 2^{\frac{1}{2}} = 10\cdot 2^{\frac{1}{2}} = 10\sqrt{2}$

Problem 42 $2\sqrt{48}-\sqrt{3} = 2\cdot(2^4\cdot 3)^{\frac{1}{2}} - 3^{\frac{1}{2}} = 2^3\cdot 3^{\frac{1}{2}} - 3^{\frac{1}{2}} = 7\sqrt{3}$

Problem 43 $-\sqrt{12}+\sqrt{75} = -(2^2\cdot 3)^{\frac{1}{2}}+(3\cdot 5^2)^{\frac{1}{2}} = -2\cdot 3^{\frac{1}{2}}+5\cdot 3^{\frac{1}{2}} = (5-2)\sqrt{3} = 3\sqrt{3}$

Problem 44 $2\sqrt{27}-\sqrt{300} = 2\cdot(3^3)^{\frac{1}{2}} - (3\cdot 10^2)^{\frac{1}{2}} = 6\cdot 3^{\frac{1}{2}} - 10\cdot 3^{\frac{1}{2}} = -4\sqrt{3}$

Problem 49 $P = 2\pi\sqrt{\frac{5.1}{32}} \approx 2.5$. Note that 32 is an approximation to gravitational acceleration.

Problem 81 Approximating:

	1.1^{10}	1.01^{100}	1.001^{1000}	1.0001^{10000}	1.00001^{100000}
	2.5937	2.7048	2.7169	2.7181	2.7183
	$0.95418e$	$0.99505e$	$0.99950e$	$0.99095e$	$1.00000e$

The constant $e = \lim_{n\to\infty}(1+\frac{1}{n})^n$, and the experimental results above bear this out. The first few digits converge very quickly.

2 Exercises 6.3

Problem 75 0.75

Problem 76 0.875

Problem 79 $0.\overline{27}$

Problem 80 $0.\overline{81}$

Problem 86 $0.105 = \frac{105}{1000} = \frac{21}{200}$

Problem 87 $0.934 = \frac{934}{1000} = \frac{467}{500}$

Problem 88 $0.7984 = \frac{7984}{10000} = \frac{499}{625}$

Problem 95

- $\frac{1}{3} = 0.\overline{3}$
- $\frac{2}{3} = 0.\overline{6}$
- $\frac{1}{3} + \frac{2}{3} = 0.\overline{3} + 0.\overline{6} = 0.\overline{9}$
- As covered in class, repeating nines are equal to a one one digit over, so $0.\overline{9} = 1$.

Problem 96 Here, $3 \cdot 0.\overline{3} = 0.\overline{9} = 1$.

3 Exercises 6.5

Problem 1 $3.00 \cdot 12 = 36$, **true**

Problem 2 $0.25 = \frac{25}{100} = \frac{1}{4}$, **true**

Problem 3 Rounding 759.367 to the second place beyond the decimal should give either 759.37 or 759.36 depending on the rounding rule. **false**

Problem 4 With round-to-nearest (even or upwards), this is **true**.

Problem 5 $0.50 = \frac{50}{100} = \frac{1}{2}$, **true**

4 Rounding and floating-point

4.1 Rounding

Round each of the following to the nearest tenth (one place after the decimal) using **round to nearest even**, **round to zero (truncation)**, and **round half-up**:

- 86.548

- 86.554
- 86.55

Number	Round to nearest even	Truncate	Round half-up
86.548	86.5	86.5	86.5
86.554	86.6	86.5	86.6
86.55	86.6	86.5	86.6

4.2 Errors in computations

Compute the following quantities with a computer or a calculator. **Write what type of computer/calculator you used and the software package if it's a computer.** Compute it as shown. Do not simplify the expression before computing it, and do not re-enter the intermediate results into the calculator or computer program. Also compute the expressions that do not include 10^{16} by hand exactly. There should be a difference between the exact result and the displayed result in some of these cases. Remember to work from the innermost parentheses outward.

- $(\overbrace{0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1}^{\text{10 times}}) - 1$
- $((\overbrace{0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1+0.1}^{\text{10 times}}) - 1) \times 10^{16}$, where 10^{16} often is entered as 1e16. If the result overflows (signals an error) on various calculators, replace 10^{16} by 10^8 in this and later portions.
- $(((2 \div 3) - 1) \times 3) + 1$
- $((((2 \div 3) - 1) \times 3) + 1) \times 10^{16}$
- $(((6 \div 7) - 1) \times 7) + 1$
- $((((6 \div 7) - 1) \times 7) + 1) \times 10^{16}$

The object of this first part is to demonstrate round-off error. The first to problems, adding 0.1 repeatedly, may see no error if the device calculates in decimal. The latter four parts should see some error regardless of the base used.

Using Octave on a 64-bit Intel-based machine with the "short" display format:

- $-1.1102 \cdot 10^{-16}$
- -1.1102
- 0 : Sometimes errors cancel themselves out. Not every computational error is **bad**.

- 0
- $-4.4409 \cdot 10^{-16}$
- -4.4409

4.3 Extra digits

Now copy down the number displayed by the first calculation in each of the following. Re-enter it as x in the second calculation.

- $1 \div 3$, then $1 \div 3 - x$ where x is the number displayed.
- If you have a calculator or program with π, π, then $\pi - x$ where x is the number displayed.

The object here is to see that the number displayed often is not the number the computer or calculator has stored.

Using Octave on a 64-bit Intel-based machine with the “short” display format:

- `1 / 3` produces `0.33333`. Then `(1/3) - 0.33333` produces `3.3333e-06` or $3.3333 \cdot 10^{-6}$.
- `pi` produces `3.1416`, and `pi - 3.1416` produces `-7.3464e-06` or $-7.3464 \cdot 10^{-6}$.