Solutions for the thirteenth week's homework
Math 131

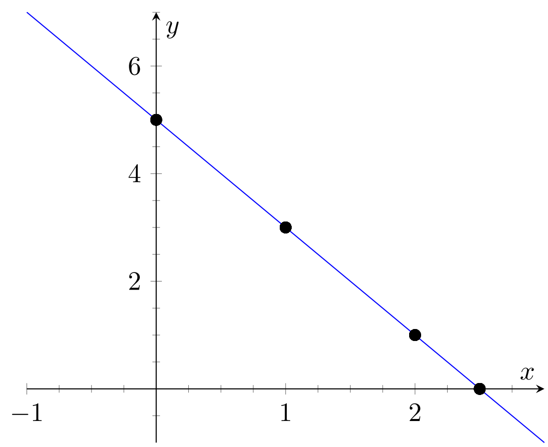
Jason Riedy

17 November, 2008

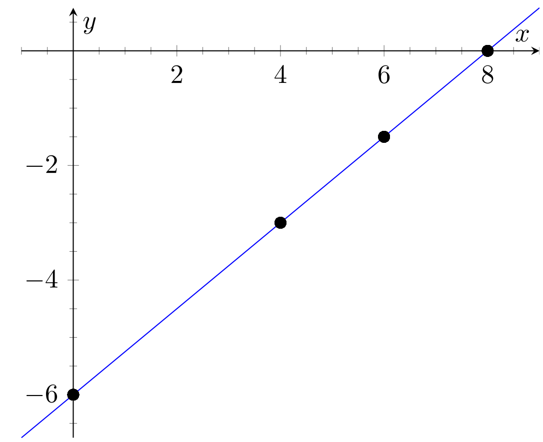
Also available as PDF.

1 Exercises for 8.2

Problem 1: $(0, 5)$, $(\frac{5}{2}, 0)$, $(1, 3)$, $(2, 1)$



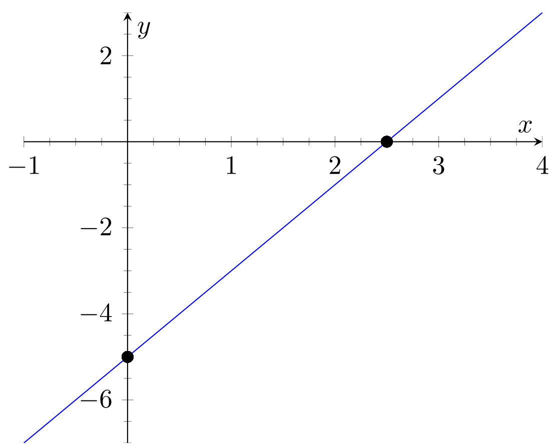
Problem 2: $(0, -6)$, $(8, 0)$, $(6, \frac{-3}{2})$, $(4, -3)$



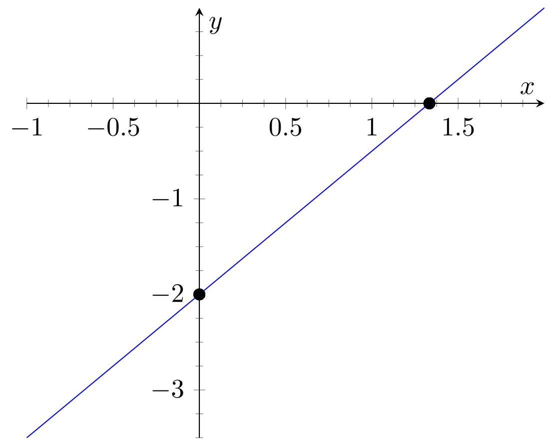
Problem 9: Substitute 0 for y, then solve for x. Or compute the intercept form.

Problem 10: Substitute 0 for x, then solve for y. Or compute the intercept form.

Problem 17: In intercept form: $\frac{x}{5/2} + \frac{y}{-5} = 1$, so the intercepts are $(\frac{5}{2}, 0)$ and $(0, -5)$.



Problem 18: In intercept form: $\frac{x}{4/3} + \frac{y}{-2} = 1$, so the intercepts are $(\frac{4}{3}, 0)$ and $(0, -2)$.



Problem 39: **Part a:** The two points are $(-1, -4)$ and $(3, 2)$. Then $\Delta x = 3 - -1 = 4$, $\Delta y = 2 - -4 = 6$, so the slope is $\frac{\Delta y}{\Delta x} = \frac{6}{4} = \frac{3}{2}$.
Part b: The two points are $(1, -2)$ and $(-3, 5)$. Then $\Delta x = -3 - 1 = -4$ and $\Delta y = 5 - -2 = 7$. The slope is $\frac{\Delta y}{\Delta x} = \frac{-7}{4}$.

Problem 57: L_1's slope is $\frac{7-6}{-8-4} = \frac{-1}{12}$. L_2's slope is $\frac{5-4}{-5-7} = \frac{-1}{12}$. The slopes are equal, so the lines are **parallel**.

Problem 58: L_1's slope is $\frac{12-15}{-7-9} = \frac{3}{16}$. L_2's slope is $\frac{5-8}{-20--4} = \frac{-3}{-16} = \frac{3}{16}$. These lines are **parallel**.

Problem 63: The "run", or change along the horizontal from the back of the deck to its fore, is $250 - 160 = 90$ feet. The "rise" is the drop from the back to the fore, or -63 feet. So the slope is $\frac{-63}{90} = \frac{-7}{10} = -0.7$. Note that if you start at the fore and face the aft, the slope will be negated (0.7). Both answers are correct so long as you explain the direction. In the end, this is a **70% grade**. Yes, that is steep.

Problem 64: A 13% grade is a slope of $\frac{13}{100}$, or 13 feet up for every 100 feet across. Given a run of 150 feet, the maximum rise is $\frac{13}{100} \cdot 150 = \frac{39}{2} = 19.5$ feet.

2 Exercises for 8.3

Problem 5: The slope is $\frac{0--3}{1-0} = 3$. With the y-intercept of -3, the slope-intercept form is $y = 3x - 3$.

Problem 6: The slope is $\frac{0--4}{2-0} = 2$. With the y-intercept of -4, the slope-intercept form is $y = 2x - 4$.

Problem 19: Starting with the form $y - y_0 = m(x - x_0)$, substituting gives $y - 8 = -2(x - 5)$. Solving for y and simplifying into slope-intercept form gives $y = -2x + 18$.

Problem 20: Starting with the form $y - y_0 = m(x - x_0)$, substituting gives $y - 10 = 1(x - 12)$. Solving for y and simplifying into slope-intercept form gives $y = x - 2$.

Problem 26: A slope of 0 is a horizontal line. Thus $y = -2$.

Problem 28: An undefined slope is a vertical line. Thus $x = -2$.

Problem 39: The points have the same y coordinate, so the line is horizontal. Thus $y = 5$ with a slope of zero.

Problem 40: The points have the same y coordinate, so the line is horizontal. Thus $y = 2$ with a slope of zero.

Problem 60: The slope of the latter line is $\frac{-2}{5}$, so $y - 1 = \frac{-2}{5}(x - 4)$ is the point-slope form. Reducing to slope-intercept form, $y = \frac{-2}{5}x + \frac{13}{5}$.

Problem 64: The slope of the latter line is $\frac{-5}{2}$. Thus a *perpendicular* line will have the slope $\frac{2}{5}$. In point-slope form, $y - -7 = \frac{2}{5}(x - 2)$, or $y = \frac{2}{5}x + \frac{-39}{5}$ in slope-intercept form.

3 Exercises for 8.7

Problem 3: $5 + 1 = 6$ and $5 - 1 = 4$, so $(5, 1)$ **is a solution**.

Problem 4: $8 - -9 = 8 + 9 = 17$ and $8 + -9 = 8 - 9 = -1$, so $(8, -9)$ **is a solution**.

Problem 15: Starting with the matrix of coefficients,

$$\begin{bmatrix} 7 & 2 & 6 \\ -14 & -4 & -12 \end{bmatrix},$$

adding twice the first row to the second produces

$$\begin{bmatrix} 7 & 2 & 6 \\ 0 & 0 & 0 \end{bmatrix}.$$

Hence both lines are the same, and the solution set consists of all points on the line $7x + 2y = 6$.

Problem 16: Here we start with

$$\begin{bmatrix} 1 & -4 & 2 \\ 4 & -16 & 8 \end{bmatrix}.$$

Subtracting four times the first row from the second yields

$$\begin{bmatrix} 1 & -4 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

Again, these lines are the same, and the solution set consists of all points on the line $x - 4y = 2$.

Problem 24: One method starts by solving the first equation for $y = 3x - 5$. Substituting into the second, $x + 2(3x - 5) = 0$ gives $7x - 10 = 0$ or $x = \frac{10}{7}$. Now substituting this x into $y = 3x - 5$ yields the solution point $(\frac{10}{7}, \frac{-5}{7})$.

Problem 25: The second equation provides $y = 2x - 1$. Then the first becomes $-x - 4(2x - 1) = -14$, or $-9x + 4 = -14$, giving $x = 2$. Then $y = 3$ and the solution is $(2, 3)$

Problem 35: Start by writing out the coefficients:

$$\begin{bmatrix} 3 & 2 & 1 & 8 \\ 2 & -3 & 2 & -16 \\ 1 & 4 & -1 & 20 \end{bmatrix}$$

Now rearrange to make cancellation easier by hand:

$$\begin{bmatrix} 1 & 4 & -1 & 20 \\ 2 & -3 & 2 & -16 \\ 3 & 2 & 1 & 8 \end{bmatrix}$$

(**Note:** If using a computer or calculator, you really should rearrange so you always divide by the largest magnitude entry remaining in the column. Here, I would not have altered the order. But this happens to be all-integer if you chose the correct operations.)

Now subtract the first row from the two remaining rows:

$$\begin{bmatrix} 1 & 4 & -1 & 20 \\ 0 & -11 & 4 & -56 \\ 0 & -10 & 4 & -52 \end{bmatrix}$$

Next, recognize that $-10 - -11 = 1$ and subtract the second row from the last row:

$$\begin{bmatrix} 1 & 4 & -1 & 20 \\ 0 & -11 & 4 & -56 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

Swap rows to place the second column's 1 in the second row:

$$\begin{bmatrix} 1 & 4 & -1 & 20 \\ 0 & 1 & 0 & 4 \\ 0 & -11 & 4 & -56 \end{bmatrix}$$

Add -11 times the second row to the last:

$$\begin{bmatrix} 1 & 4 & -1 & 20 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 4 & -12 \end{bmatrix}$$

Now divide the last by four:

$$\begin{bmatrix} 1 & 4 & -1 & 20 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

Eliminate along the last column by adding the last row to the first:

$$\begin{bmatrix} 1 & 4 & 0 & 17 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

And eliminate the rest of the second column by subtracting four times the second row from the first:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

This gives the solution $(\mathbf{1}, \mathbf{4}, \mathbf{-3})$.

Problem 36: Begin with:

$$\begin{bmatrix} -3 & 1 & -1 & -10 \\ -4 & 2 & 3 & -1 \\ 2 & 3 & -2 & -5 \end{bmatrix}$$

Again, if you chose the order correctly, all arithmetic will be with integers. But this time I'll perform the operations in the sensible numerical order. First, swap the first two rows to place the largest magnitude entry, -4, at the top:

$$\begin{bmatrix} -4 & 2 & 3 & -1 \\ -3 & 1 & -1 & -10 \\ 2 & 3 & -2 & -5 \end{bmatrix}$$

Now divide the first row by four:

$$\begin{bmatrix} 1 & -0.5 & -0.75 & 0.25 \\ -3 & 1 & -1 & -10 \\ 2 & 3 & -2 & -5 \end{bmatrix}$$

Subtract multiples of the first row from the others (-3 for the second row, 2 for the third):

$$\begin{bmatrix} 1 & -0.5 & -0.75 & 0.25 \\ 0 & -0.5 & -3.25 & -9.25 \\ 0 & 4 & -0.5 & -5.5 \end{bmatrix}$$

Swap the second and third rows:

$$\begin{bmatrix} 1 & -0.5 & -0.75 & 0.25 \\ 0 & 4 & -0.5 & -5.5 \\ 0 & -0.5 & -3.25 & -9.25 \end{bmatrix}$$

Divide the second row by four:

$$\begin{bmatrix} 1 & -0.5 & -0.75 & 0.25 \\ 0 & 1 & -0.125 & -1.375 \\ 0 & -0.5 & -3.25 & -9.25 \end{bmatrix}$$

Subtract -0.5 times the second row from the third:

$$\begin{bmatrix} 1 & -0.5 & -0.75 & 0.25 \\ 0 & 1 & -0.125 & -1.375 \\ 0 & 0 & -3.3125 & -9.9375 \end{bmatrix}$$

Divide the third row by -3.3125:

$$\begin{bmatrix} 1 & -0.5 & -0.75 & 0.25 \\ 0 & 1 & -0.125 & -1.375 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Subtract -0.125 times the third row from the second:

$$\begin{bmatrix} 1 & -0.5 & -0.75 & 0.25 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Now subtract -0.5 times the second row and -0.75 times the third row from the first:

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

This gives the solution $(2, -1, 3)$.

Note: Solving that last problem in an environment like Octave is slightly easier:

```
octave> [-3 1 -1; -4 2 3; 2 3 -2] \ [-10; -1; -5]
ans =

   2
  -1
   3
```

There is a **lot** of work in making that magic \ operator function correctly.

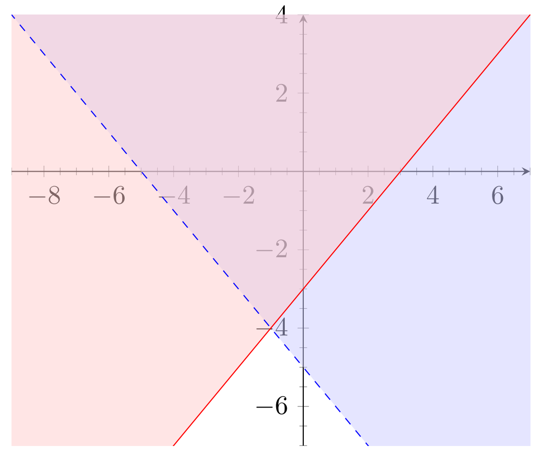
4 Exercises for 8.8

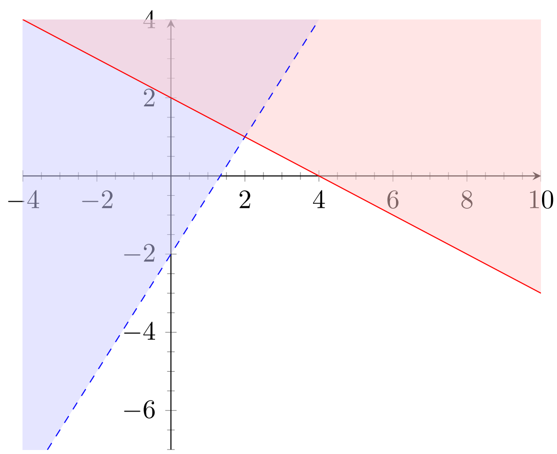
Problem 1: **C**: Dashed lines denote $<$ and $>$.

Problem 2: **A**: Solid lines denote $\leq$ and $\geq$.

Problem 3: **B**

Problem 4: **D**

Problem 21: 

Problem 22: 

Problem 25: The interesting vertices are the x- and y-intercepts along with the intersections of those lines, along with the origin $(0, 0)$. The intersection is at $(1.2, 1.2)$. All values of $5x + 2y$ at these points:

Point	Value
$(0, 0)$	0
$(0, 2)$	4
$(3, 0)$	15
$(0, 6)$	12
$(1.5, 0)$	7.5
$(1.2, 1.2)$	8.4

We could draw the graph to determine which vertices are in the feasible region. Alternately, we can test points against all the inequalities, beginning with the largest value and working downwards. The point $(3, 0)$ does not satisfy $4x + y \leq 6$ and is not feasible. The point $(0, 6)$ does not satisfy $2x + 3y \leq 6$ and is not feasible. The point $(1.2, 1.2)$ satisfies all the inequalities, so the **largest value is 8.4 occuring at** $(\mathbf{1.2}, \mathbf{1.2})$.

Problem 26: Listing all of the interesting points only once gives

Point	Value
$(0, 0)$	0
$(10, 0)$	10
$(4, 0)$	4
$(0, 10)$	30
$(20/3, 10/3)$	50/3
$(10/3, 5/3)$	25/3

Now check points from the least value upwards. $(0, 0)$ does not satisfy $5x + 2y \geq 20$. $(4, 0)$ does not satisfy $2y \geq x$. $(10/3, 5/3)$ satisfies every constraint. Thus **the minimal value is 25/3 at point** $(\mathbf{10/3}, \mathbf{5/3})$**.**