Solutions for the fourteenth week's homework Math 131

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Also available as PDF.

1 Exercises for 7.3

Problem 25 The ratio is $\frac{2.5\text{ oz oil}}{1\text{ gal gas}}$. So given 2.75 gallons, $\frac{2.5\text{ oz oil}}{1\text{ gal gas}} \cdot 2.75\text{ gal gas} = \mathbf{6.875}$ **ounces** of oil are required.

Problem 26 Here the ratio is $\frac{5.5\text{ oz oil}}{1\text{ gal gas}}$. Given 22 ounces of oil, $\frac{1\text{ gal gas}}{5.5\text{ oz oil}} \cdot 22\text{ oz oil} = \mathbf{4}$ **gallons** of gas are required.

Problem 68 "Varies directly" implies a proportional relationship. Here the ratio is $\frac{5\text{ psi}}{200\text{ deg K}}$. So at 300 degrees Kelvin, the pressure is $\frac{5\text{ psi}}{200\text{ deg K}} \cdot 300\text{ deg K} = \mathbf{7.5}$ **psi**.

Problem 70 The correct ratio here is $\frac{12\text{ pounds}}{3\text{ in}}$, and the force to compress 5 inches is $\frac{12\text{ pounds}}{3\text{ in}} \cdot 5\text{ in} = \mathbf{20}$ **pounds**.

2 Exercises for 7.4

Problem 64 The target heart rate for a 35 year old lies in $[129.5, 157.25]$. Rounding this to nearest **even** (safer in general) gives the interval $[\mathbf{130}, \mathbf{157}]$. When dealing with intervals like these, however, rounding each endpoint to nearest is not necessarily the best idea. In this case, the result was *within* the original interval, so all numbers in the rounded interval still satisfy the relationship. If we had rounded 129.5 to 129, then the lower endpoint would not have satisfied the relationship. Which direction to round depends on the problem and assumptions in the model[1]. **Your age:** Likely is less than mine, which in turn is less than the first part. Note that you can treat each side, $.7(220 - A)$ and $.85(220 - A)$, as a line. Both have negative slopes, so both *decrease* with increasing age.

[1] See the IEEE interval standardization group at `http://www.cs.utep.edu/interval-comp/standard.html` for links to more information.

Problem 65 At break-even, the cost C is equal to the revenue R. So at break-even, $20x + 100 = 24x$ and $x = 25$. Now the question becomes on which side the company shows a *profit*, or $R - C > 0$. Substituting for R and C, $(24x) - (20x + 100) > 0$ or $x > 25$. The smallest whole number of units x to show a profit is then **26** and *not* 25.

Problem 66 Here $R = 5.5x$ and $C = 3x + 2300$, so $R - C > 0$ becomes $5.5x - (3x + 2300) > 0$ or $2.5x > 2300$ and $x > 920$. So the smallest profitable x is **921**.

3 Exercises for 7.5

When what I say clearly does *not* apply to the problem, you should tell me. Here I assigned Problem 60 in Section 7.5 when I meant Section 7.7. I've included the answer for the problem in Section 7.5, even though it's a pointless problem.

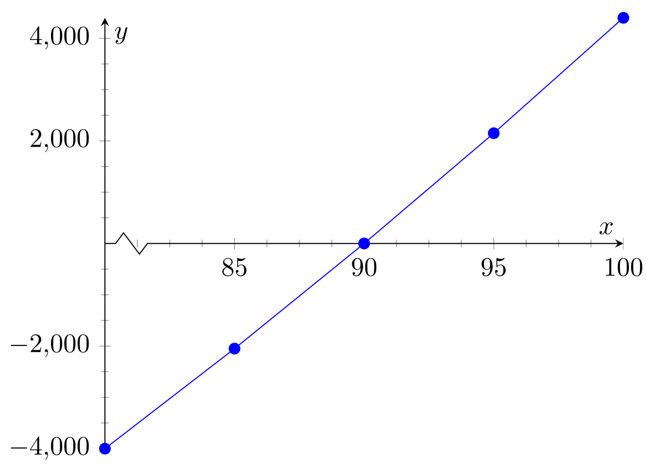
Problem 60 $(p^{-1})^3 p^{-4} = p^{-3} p^{-4} = \mathbf{p^{-7}}$

4 Exercises for 7.7

Problem 60 by plotting The function to plot is $y = x^2 + (x + 30)^2 - 150^2 = 2x^2 + 60x - 21600$. Evaluating at the points $x \in \{80, 85, 90, 95, 100\}$ gives:

x	80	85	90	95	100
y	-4000	-2050	0	2150	4400

So the zero is at $x = 90$. Plotting these segments:



Problem 60 by Pythagorean thm By the Pythagorean theorem, $x^2 + (x + 30)^2 = 150^2$. Then we need to find roots of $2x^2 + 60x - 21600 = 2(x^2 + 30x - 10800)$. We can solve this by simply trying points or by the quadratic equation. For the latter, we can use the simpler $x^2 + 30x - 10800$ and find

$$\begin{aligned} x &= \frac{-30 \pm \sqrt{30^2 + 4 \cdot 10800}}{2} \\ &= \frac{-30 \pm 210}{2} = -15 \pm 105 \\ &= 90 \text{ or } -120. \end{aligned}$$

For using points, find convenient numbers in the problem and try them. Here, to numbers that pop out are 30 and 150. Evaluating at each gives $(30, -14400)$ and $(150, 50400)$, so we know the zero must be somewhere between them. Half-way is $(30 + 150)/2 = 90$, so trying 90 finds the solution.

5 Exercises for 8.1

Problem 56, using the point form of the line The two points closest to 1985 are $(1980, 4.5)$ and $(1990, 5.2)$. The point form here is

$$\frac{x - 1980}{1990 - 1980} = \frac{y - 4.5}{5.2 - 4.5}.$$

Plugging in $x = 1985$ and solving for y gives $y = \mathbf{4.85}$ **million**. Repeating around 1995,

$$\frac{x - 1990}{2000 - 1990} = \frac{y - 5.2}{5.8 - 5.2}.$$

Using $x = 1995$ gives $y = \mathbf{5.5}$ **million**.

6 Exercises for 8.3

Problem 70 *(This was the "bad graph" example I used previously to show how a linear relationship can be hidden.* We can start with the point form,

$$\frac{x - 150}{1400 - 150} = \frac{y - 5000}{24000 - 5000}.$$

Solving for y gives the slope-intercept form,

$$y = 15.2x + 2720.$$

To verify this, check that $x = 150$ gives $y = 5000$, and $x = 1400$ gives $y = 24000$.

Problem 72 Part a Again, starting from point form is easiest:

$$\frac{x-5}{7-5} = \frac{y-24075}{26628-24075}.$$

Solving for y,

$$\begin{aligned} y &= 24075 + \frac{2553}{2}(x-5) \\ &= 1276.5x + 17692.5. \end{aligned}$$

Part b The slope from 1995 to 1997 is positive, but the graph shows a negative slope from 1993 or 1994 to 1995. Thus the linear model above will not approximate those well. The graph shows non-linear variation, so there is no reason to expect the 1995–1997 line to continue to 1998, so **no** to all.

Problem 74 Part a and b The points in question are $(\mathbf{0}, \mathbf{32})$ and $(\mathbf{100}, \mathbf{212})$.

Part c The slope is $\frac{\Delta y}{\Delta x} = \frac{212°\mathrm{F}-32°\mathrm{F}}{100°\mathrm{C}-0°\mathrm{C}} = \frac{\mathbf{9}°\mathrm{F}}{\mathbf{5}°\mathrm{C}}$.

Part d The point $(0, 32)$ provides the y-intercept, so the line is $\mathbf{y} = \frac{\mathbf{9}°\mathrm{F}}{\mathbf{5}°\mathrm{C}}\mathbf{x} + \mathbf{32}°\mathrm{F}$.

Part e A function of x in terms of y: $\mathbf{x} = \frac{\mathbf{5}°\mathrm{C}}{\mathbf{9}°\mathrm{F}}(\mathbf{y} - \mathbf{32}°\mathrm{F})$.

Part f The graph shows that 50°C is 122°F. You can tell that the graph is of the conversion °C → °F because the y-intercept is positive.

7 Exercises for 8.6

Problem 50 At 7% compounded quarterly[2], \$60 000 will grow to in \$60 000 $\cdot (1 + \frac{0.07}{4})^{20} \approx$ \$84886.69 5 years. Compounded "continuously"[3], the same amount will grow to \$60 000 $\cdot\, e^{0.0675 \cdot 5} \approx$ \$84086.38. Here the higher rate compounded quarterly is the better one, with a difference of *approximately* \$800.32.

8 Exercises for 8.7

Problem 50 The nonsensical system here is $W = L - 44$ ft and $2L + 2W = 288$ ft. Solving by substituting the former into the latter gives $L = 144\text{ ft} - W = 144\text{ ft} - L + 44\text{ ft}$, so $L = \mathbf{94\ ft}$. Now $W = L - 44\text{ ft} = \mathbf{50\ ft}$.

[2] Note that the text's definition of "compounded quarterly" is not used by all financial institutions. Always check how your institution defines their terms. Also, intermediate quantities in the calculation may be rounded according to local laws and the institution's rules. Yes, it really is this complicated.

[3] Again, check with individual institutions about their definitions.

Problem 78 The first statement gives $\frac{150 \text{ km}}{T \text{ km/hr}} = \frac{400 \text{ km}}{P \text{ km/hr}}$ or equivalently $150 \text{ km} \cdot P \text{ km/hr} = 400 \text{ km} \cdot T \text{ km/hr}$. The second statement gives $P \text{ km/hr} = 3T \text{ km/hr} - 20 \text{ km/hr}$. Solving, the plane's speed is T km/hr = **60 km/hr**, and the train's speed is P km/hr = **160 km/hr**. Plugging these into either of the given equations verifies the solution.

Problem 86 Translating into algebra,

$$\begin{aligned} C &= A + B + 10, \\ B &= 2A, \text{ and} \\ A + B + C &= 490. \end{aligned}$$

This is best solved by substitution. Written in terms of C, the last equation becomes $(C - 10) + C = 490$, or $\mathbf{C} = \mathbf{250}$. Now writing the first in terms of A after substituting C, $250 = A + 2A + 10$ or $\mathbf{A} = \mathbf{80}$. Now $\mathbf{B} = \mathbf{160}$. Substituting these into the equations above verifies this solution.

9 Exercises for 8.8

Problem 30 Let A and B be the servings of each product. Summarizing the information:

Product	I (g/serving)	II (g/serving)	cost (\$/serving)
A	3	2	0.25
B	2	4	0.40

So the function to minimize is the cost $0.25A + 0.40B$. The constraints are that there be at least 15 g of I, or $3A + 2B \geq 15$, and 15 g of II, or $2A + 4B \geq 15$. There also are trivial constraints $A \geq 0$ and $B \geq 0$. The problem to solve is to

$$\begin{aligned} \min_{A,B} \quad & 0.25A + 0.40B \\ \text{subject to} \quad & 3A + 2B \geq 15, \\ & 2A + 4B \geq 15, \\ & A \geq 0, \text{ and} \\ & B \geq 0. \end{aligned}$$

The first two lines intersect at $(3.75, 1.875)$. The A intercepts are 5 and 7.5, and only $(0, 7.5)$ is feasible. The B intercepts are 7.5 and 3.75, and only $(7.5, 0)$ is feasible. So the points to check and the function values are

Point	Cost
$(3.75, 1.875)$	\$1.6875
$(0, 7.5)$	\$3
$(7.5, 0)$	\$1.875

So the cheapest combination is 3.75 of A and 1.875 of B at \$1.6875.

Problem 34 Summarizing and assigning variables:

Kind	Variable	Oven time (hr)	Decorating time (hr)	Profit (\$)
Cookie	C	1.5	$\frac{2}{3}$	20
Cake	A	2	3	30

So the problem is to

$$\begin{aligned} \max_{C,A} \quad & 20C + 30A \\ \text{subject to} \quad & 1.5C + 2A \leq 15, \\ & \frac{2}{3}C + 3A \leq 13, \\ & C \geq 0, \text{ and} \\ & A \geq 0. \end{aligned}$$

The interesting points and their values:

Point	Profit (\$)
$(0, 0)$	0
$(6, 3)$	210
$(0, \frac{13}{3})$	130
$(10, 0)$	200

So the best combination is **6 cookie batches and 3 cake batches** for a **profit of \$210**.