Solutions for the sixth week's homework
Math 131

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Also available as PDF.

1 Section 4.1, problems 35 and 36

Multiplying $26 \cdot 53$ by the Egyptian algorithm:

1	53
2	106
4	212
8	424
16	848

Now $26 = 16 + 8 + 2$, so $26 \cdot 53 = 848 + 424 + 106 = \mathbf{1378}$.

Computing $33 \cdot 81$:

1	81
2	162
4	324
8	648
16	1296
32	2592

Because $33 = 32 + 1$, $33 \cdot 81 = 2592 + 81 = \mathbf{2673}$.

2 Section 4.2

Problem 2 $925 = 9 \cdot 10^2 + 2 \cdot 10^1 + 5 \cdot 10^0$

Problem 3 $3774 = 3 \cdot 10^3 + 7 \cdot 10^2 + 7 \cdot 10^1 + 4 \cdot 10^0$

Problem 5 $4 \cdot 10^3 + 9 \cdot 10^2 + 2 \cdot 10^1 + 4 \cdot 10^0$

Problem 6 $5 \cdot 10^4 + 2 \cdot 10^3 + 1 \cdot 10^2 + 1 \cdot 10^1 + 8 \cdot 10^0$

Problem 11 6209

Problem 12 503568

3 Section 4.3

Problem 2 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24

Problem 7 $\mathbf{B6E_{16}}$, $\mathsf{B6F}_{16}$, $\mathbf{B70_{16}}$

Problem 8 $\mathbf{10110_2}$, 10111_2, $\mathbf{11000_2}$

Problem 19 $\mathsf{3BC}_{16} = (3 \cdot 16 + 11) \cdot 16 + 12 = 956$

Problem 20 $34432_5 = (((3 \cdot 5 + 4) \cdot 5 + 4) \cdot 5 + 3) \cdot 5 + 2 = 2492$

Problem 21 $2366_7 = ((2 \cdot 7 + 3) \cdot 7 + 6) \cdot 7 + 6 = 881$

Problem 22 $101101110_2 = (((((((1 \cdot 2 + 0) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 0) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 0 = 366$

Problem 37 $586 = 512 + 64 + 8 + 2 = 2^9 + 2^6 + 2^3 + 2^1 = 1001001010_2$

Problem 38 $12888 = 3 \cdot 4096 + 1 \cdot 512 + 1 \cdot 64 + 3 \cdot 8 = 3 \cdot 8^4 + 1 \cdot 8^3 + 1 \cdot 8^2 + 3 \cdot 8 = 31130_8$

Problem 39 $8407 = 102112101_3$

Problem 40 $11028 = 2230110_4$

Problem 57 $9 \cdot 12^2 + 10 \cdot 12 + 11 = 1427$

4 Positional form

> Take a familiar incomplete integer, _679_, and express it as a sum of the digits times powers of ten using variables x_0 and x_4 for the digits in the blanks. Simplify to the form of $x_4 \cdot 10^4 + x_0 \cdot 10^0 + z$, where z is a single number in positional form (a sequence of digits). Does 72 divide z? Does 8 divide z? Does 9 divide z? Remember that $72 = 8 \cdot 9$. We will use this example again in the next chapter.

We can expand _679_ to be $x_4 \cdot 10^4 + 6 \cdot 10^3 + 7 \cdot 10^2 + 9 \cdot 10 + x_0 = x_4 \cdot 10^4 + x_0 + 6790$, so $\mathbf{z} = \mathbf{6790}$. Unfortunately, none of the numbers provided divide cleanly into 6790. Jumping to the next chapter, $6790 = 94 \cdot 72 + 22 = 848 \cdot 8 + 6 = 754 \cdot 9 + 4$.

5 Operations

Multiplication:

- $47 \cdot 3 = (4 \cdot 10 + 7) \cdot 3 = 12 \cdot 10 + 21 = 141$.
  Or in table form:

$$\begin{array}{rr} & 47 \\ \cdot & 3 \\ \hline & 21 \\ & 12 \\ \hline & 141 \end{array}$$

- $47 \cdot 13 = (4 \cdot 10 + 7) \cdot (10 + 3) = (4 \cdot 10^2 + 7 \cdot 10) + (12 \cdot 10 + 21) = 470 + 141 = 611$.
  Or in table form:

$$\begin{array}{rr} & 47 \\ \cdot & 13 \\ \hline & 21 \\ & 12 \\ \hline & 141 \\ & 47 \\ \hline & 611 \end{array}$$

- $47 \cdot 23 = (4 \cdot 10 + 7) \cdot (2 \cdot 10 + 3) = (8 \cdot 10^2 + 14 \cdot 10) + (12 \cdot 10 + 21) = 940 + 141 = 1081$.
  Or in table form:

$$\begin{array}{rr} & 47 \\ \cdot & 23 \\ \hline & 21 \\ & 12 \\ \hline & 141 \\ & 14 \\ \hline & 240 \\ & 8 \\ \hline & 1081 \end{array}$$

Addition:

- $47 + 52 = (4 + 5) \cdot 10 + (7 + 2) = 9 \cdot 10 + 9 = 99$.
- $47+53 = (4+5) \cdot 10+(7+3) = \mathbf{9 \cdot 10 + 10} = \mathbf{10 \cdot 10 + 0} = 1 \cdot 10^2 + 0 \cdot 10 + 0 = 100$. Both bold forms are redundant intermediate representations.
- $47+54 = (4+5) \cdot 10+(7+4) = \mathbf{9 \cdot 10 + 11} = \mathbf{10 \cdot 10 + 1} = 1 \cdot 10^2 + 0 \cdot 10 + 1 = 101$. Both bold forms are redundant intermediate representations.

Subtraction:

- $7-19=(0-1)\cdot 10+(7-9)=\mathbf{-1\cdot 10+-2\cdot}=-1\cdot(1\cdot 10+2)=-12$. The bold form is a redundant intermediate representation.
- $19-19=(1-1)\cdot 10+(9-9)=0+0=0$.
- $20-19=(2-1)\cdot 10+(0-9)=\mathbf{1\cdot 10+-9}=0\cdot 10+(10-9)=1$. The bold form is a redundant intermediate representation.
- $29-19=(2-1)\cdot 10+(9-9)=1\cdot 10+0=10$.