

Solutions for the eighth week's homework

Math 131

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Also available as PDF.

1 Exercises 5.3

Problem 62 If $(p, q) = p$, then $p \mid q$ and q is a multiple of p .

Problem 66 We want the least common multiple of 6 and 10. The nights off intersect every 30 days. July has 31 days, so by this method their next shared day off is the 31st. On the “trick question” side, though, they may have July 4th off together...

Problem 70 Here we need the greatest common divisor, $(60, 72) = 12$. So the longest common length is 1 foot.

2 Computing GCDs

Compute the following using **both** the prime factorization method and the Euclidean algorithm:

- $(720, 241)$
- $(64, 336)$
- $(-15, 75)$

Prime factorizations:

- 241 is prime. So $(720, 241) = 1$.
- $64 = 2^6$, $336 = 2^4 \cdot 3 \cdot 7$. $(64, 336) = 2^4 = 16$.
- $(-15, 75) = (15, 75)$. $15 = 3 \cdot 5$, $75 = 3 \cdot 5^2$, so $(-15, 75) = 15$.

Euclidean algorithm:

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$$\begin{aligned} 720 &= 2 \cdot 241 + 238, \\ 241 &= 1 \cdot 238 + 3, \\ 238 &= 79 \cdot 3 + \mathbf{1} \\ 3 &= 3 \cdot 1 + 0. \end{aligned}$$

So $(720, 241) = \mathbf{1}$.

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$$\begin{aligned} 336 &= 5 \cdot 64 + \mathbf{16}, \\ 64 &= 4 \cdot 16 + 0. \end{aligned}$$

So $(336, 64) = 16$.

- $(-15, 75) = (15, 75)$:

$$75 = 5 \cdot \mathbf{15} + 0.$$

So $(-15, 75) = 15$.

3 Computing LCMs

Compute the least common multiples:

- $\text{lcm}(64, 336)$
- $\text{lcm}(11, 17)$
- $\text{lcm}(121, 187)$
- $\text{lcm}(2025, 648)$
- $\text{lcm}(64, 336) = 64 \cdot 336 / (336, 64) = 1344$
- Both are prime, so $\text{lcm}(11, 17) = 11 \cdot 17 = 187$
- $\text{lcm}(121, 187) = \text{lcm}(11^2, 11 \cdot 17) = 11^2 \cdot 17 = 2057$
- $\text{lcm}(2025, 648) = \text{lcm}(3^3 \cdot 5^2, 2^3 \cdot 3^4) = 2^3 \cdot 3^4 \cdot 5^2 = 16200$