Solutions for the ninth week's homework
Math 131

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20 October, 2008

Also available as PDF.

1 Linear Diophantine equations

Find **two** integer solutions to each of the following, or state why no solutions exist:

- $64x + 336y = 32$
- $33x - 27y = 11$
- $31x - 27y = 11$

- From a previous problem, we have that $336 = 64 \cdot 5 + 16$. Thus $336 \cdot 1 + 64 \cdot -5 = 16$ and $336 \cdot 2 + 64 \cdot -10 = 32$. So one solution is $\mathbf{x_0 = -10}$ and $\mathbf{y_0 = 2}$. The general solution is $x = x_0 + t \cdot 336/(336, 64) = -10 + 21t$ and $y = y_0 - t \cdot 64/(336, 64) = 2 - 4t$ for any integer t. Another solution then is $\mathbf{x(1) = -10 + 1 \cdot 21 = 11}$ and $\mathbf{y(1) = 2 - 4 \cdot 1 = -2}$.
- Here, $(33, 27) = (3 \cdot 11, 3^3) = 3$. Now $3 \nmid 11$, so there are **no solutions**.
- Now 31 is prime, so $(31, 27) = 1 \mid 11$ and there are solutions. Running through the Euclidean algorithm we see that

$$\begin{aligned} 31 &= 27 \cdot 1 + 4, \\ 27 &= 4 \cdot 6 + 3, \text{and} \\ 4 &= 3 \cdot 1 + 1. \end{aligned}$$

Starting from the bottom and substituting for the previous remainder,

$$\begin{aligned} 4 + 3 \cdot (-1) &= 1, \\ 4 + (27 + 4 \cdot (-6)) \cdot -1 = 27 \cdot (-1) + 4 \cdot 7 &= 1, \\ 27 \cdot (-1) + (31 + 27 \cdot (-1)) \cdot 7 = 31 \cdot 7 + 27 \cdot (-8) &= 1. \end{aligned}$$

We find that $31 \cdot 7 + 27 \cdot (-8) = 1$, so $31x - 27y = 11$ has an initial solution of $\mathbf{x_0 = 7 \cdot 11 = 77}$ and $\mathbf{y_0 = -1 \cdot -8 \cdot 11 = 88}$.

The general solutions have the form

$$x = x_0 + t\frac{-27}{(31,27)} = 77 - 27t, \text{ and}$$
$$y = y_0 - t\frac{31}{(31,27)} = 88 - 31t,$$

Another solution is given by $\mathbf{x(1) = 77 - 27 \cdot 1 = 50}$ and $\mathbf{y(1) = 88 - 31 \cdot 1 = 57}$.

2 Exercises 6.3

Problem 5 $\frac{16}{48} = \frac{16 \cdot 1}{16 \cdot 3} = \frac{1}{3}$

Problem 6 $\frac{21}{28} = \frac{7 \cdot 3}{7 \cdot 4} = \frac{3}{4}$

Problem 9 $\frac{3}{8} = \frac{5 \cdot 3}{5 \cdot 8} = \frac{15}{40}$, $\frac{3}{8} = \frac{-1 \cdot 3}{-1 \cdot 8} = \frac{-3}{-8}$, $\frac{3}{8} = \frac{2 \cdot 3}{2 \cdot 8} = \frac{6}{16}$

Problem 10 $\frac{9}{10} = \frac{-2 \cdot 9}{-2 \cdot 10} = \frac{-18}{-20}$, $\frac{9}{10} = \frac{2 \cdot 9}{2 \cdot 10} = \frac{18}{20}$, $\frac{9}{10} = \frac{11 \cdot 9}{11 \cdot 10} = \frac{99}{110}$

Problem 13
- $\frac{2}{6} = \frac{1}{3}$
- $\frac{1}{4}$
- $\frac{4}{10} = \frac{2}{5}$
- $\frac{3}{9} = \frac{1}{3}$

Problem 14
- $\frac{12}{24} = \frac{1}{2}$
- $\frac{6}{24} = \frac{1}{4}$
- $\frac{12}{16} = \frac{3}{4}$
- $\frac{2}{16} = \frac{1}{8}$

Problem 20 $\frac{8}{9}$

Problem 22 $\frac{41}{90}$

Problem 24 $\frac{14}{60} = \frac{7}{30}$

Problem 26 $\frac{41}{60}$

Problem 28 $\frac{3}{28}$

Problem 30 $\frac{-1}{6}$

Problem 32 $\frac{1}{4}$

Problem 34 $\frac{-3}{10}$

Problem 36 $\frac{-3}{20}$

Problem 39 $\frac{13}{3}$

Problem 40 $\frac{31}{8}$

Problem 57

$$\begin{aligned} 2+\frac{1}{1+\frac{1}{3+\frac{1}{2}}} &= 2+\frac{1}{1+\frac{2}{7}} \\ &= 2+\frac{7}{9} \\ &= \boldsymbol{\frac{25}{9}} \end{aligned}$$

Problem 58

$$\begin{aligned} 4+\frac{1}{2+\frac{1}{1+\frac{1}{3}}} &= 4+\frac{1}{2+\frac{3}{4}} \\ &= 4+\frac{4}{11} \\ &= \boldsymbol{\frac{48}{11}} \end{aligned}$$