

# Math 202 notes

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*Notes also available as PDF.*

## 1 Review

### 1.1 Problem solving

- Goal is to build up mathematical “common sense”

## 1.2 Categories

- Problems to find
- Problems to prove

## 2 Today's goal: Problem solving principles

- Review the principles
- Two tactics: Guessing and tabling.

## 3 Pólya's principles

These are **principles** and not a recipe. Use these to form a problem-solving plan. (problem solving as a problem...)

- Understand the problem
- Divise a plan
  - We will explore a taxonomy of plans
- Carry out the plan
- Examine the solution

### 3.1 Understand the problem

- Often helps to rephrase a few ways.
  - In English (or whatever is appropriate)
  - With mathematical notation
- Determine what may be relevant.
- Sketch the problem graphically, with numbers, with physical items... Whatever works for **you** on this problem.
- Decide if the problem may **have** a reasonable solution.

### 3.2 Divise a plan

- Taxonomy of plans in the text.
- Will explore shortly.

- The last “be ingenious”... Don’t take it seriously. Far more work is accomplished by being systematic.
- The “ingenious” part comes with practice, building up connections in your mind.  
Sure I’m lucky. And the more I practice, the luckier I get. – Gary Player, golfer
- One goal: Turn a strategy into a list of questions to ask students.
- When I think about the strategy, I consider what I want to write about it. A major goal of mathematics is communicating ideas well.

### 3.3 Carry out the plan

- Details depend on the plan...
- May work, may not work.
- Dead-ends are common when approaching new styles of problems.
- Moving through the plan requires attention to detail.
  - Mathematicians and scientists grow to **loathe** +/- signs.
  - Eventually, you learn which details can be “fixed” later.

### 3.4 Examine your solution

- Very, very important.
- Check your results somehow, possibly by varying the problem a little.
- Try to generalize a little.
- **Interpret** your results. Often provides a check in itself, or leads to an alternate route.
  - Common in mathematics: First publication of a result is long and hairy. Interested people being interpreting it, and a short or more direct proof is found.
  - Erdős and the “book proof”.

## 4 Two closely related tactics, guessing and making a list

- Both are forms of **searching**

- Guessing: heuristic
- Tabling: methodical

## 4.1 Guessing

- Begins as a hunch
- A few examples later becomes a guess
- More examples (fitting into a mathematical dialog) becomes a conjecture.

## 4.2 Example 1.3 from the text

- Note: Phrases like “pretty clear” and “obvious” are dangerous.
- First problem to solve: What is the problem?
  - Want to extend the pattern. What is the pattern?
- Rephrase the problem. Variables?
  - Not for elementary students, but it helps to stay ahead of them...
- Before guessing **look for relationships**. A little larger here a little smaller there.
- Now start guessing and checking.
  - Use the relationships, and look for more. This helps target your guesses.
  - That is the search...
- Now have two examples. Look back at what we’ve done, and look ahead to what we can do.
  - Look for other relationships.
  - Construct another problem with a solution

## 4.3 Tabling / Making a list

- Search through the space of answers methodically.
- Ensures you won’t miss anything.
- Blind table construction can be **long**...

#### 4.4 Example 1.4 from the text: a counting problem.

When you read it, note the follow,

- The technique is to be methodical in constructing the table.
- Take care to chose **one** method and follow it.
- If you remember truth tables from logic, same idea.

Example 1.4, page 15 (?):

##### **Make an orderly list**

How many different total scores could you make if you hit the dartboard shown with three darts?

(Three nested circles. Scores 10 for the innermost, 5 for the middle, and 1 for the outer ring.)

##### **Understand the problem**

Three darts hit the dartboard and each scores a 1, 5, or 10. The total score is the sum of the scores for the three darts. There could be three 1s, two 1s and a 5, one 5 and two 10s, and so on. The fact that we are told to find the total score when throwing three darts is just a way of asking what sums can be made using three numbers, each of which is either 1, 5, or 10.

##### **Devise a plan**

If we just write down sums hit or miss, we will almost surely overlook some of the possibilities. Using an orderly scheme instead, we can make sure that we obtain all possible scores. Let's make such a list. We first list the score if we have three 1s. then two 1s and one 5, then two 1s and no 5s, and so on. In this way, we can be sure that no score is missed.

##### **Carry out the plan**

Number of 1s	Number of 5s	Number of 10s	Total Score
3	0	0	3
2	1	0	7
2	0	1	12
1	2	0	11
1	1	1	16
1	0	2	21
0	3	0	15
0	2	1	20
0	1	2	25
0	0	3	30

The possible total scores are listed.

### Look back

Here the key to the solution was in being very systematic. We were careful first to obtain all possible scores with three 1s, then two 1s, then no 1s. With two 1s there could be either a 5 or a 10 as shown. For one 1 the only possibilities are two 5s and no 10s, one 5 and one 10, or no 5s and two 10s. Constructing the table in this orderly way makes it clear that we have not missed any possibilities.

My additional notes:

Note the similarity with numbers. Consider listing all numbers that fit the following two properties:

1. Use only the digits 0, 1, 2, and 3.
2. Have at most three digits.

Then drop those numbers whose digits do not add to three.

That's another way to construct a table like this: Consider a larger table where it is easier to be systematic, then remove numbers that do not fit the problem.

## 4.5 Different example to show bisection

Modified problem 12.a from the text's chapter review exercises.

**Geometric progression** Sequence of numbers defined by a starting number and a constant. The second number is generated by multiplying by the constant, the third by multiplying again.

Consider the sequence where 3 is the starting number and two is the constant.

- First:  $3 = 3$
- Second:  $6 = 3 \cdot 2$
- Third:  $12 = 6 \cdot 2 = 3 \cdot 2^2$
- and so forth.

Which term in the sequence is 768?

- Solve by a list? Could be long.
- What is  $2^{10}$ ? What is  $3 \cdot 2^{10}$ ?
- Know third and 11th term. Third is smaller, 11th is larger.
- Which term to try next?
- Half-way is the 7th term:  $2^6 \cdot 3 = 192$

- Now what region? ( $> 7$ th,  $< 11$ th)
- 9th: 768. **DONE**
- Calculated **three** terms (11th, 7th, 9th) rather than **five**.

## 5 Next time: More problem solving ideas.

## 6 Homework

Groups are fine, turn in your own work. Homework is due in or before class on Mondays.

Write out (briefly) your approach to each problem.

- Problem set 1.1:
  - Problems 10, 13 (Hint: find a way to make a smaller table)
- Construct an example like 1.3 where there is **no** solution. Explain what lead you to its construction. (Hint:
- Problem set 1.2:
  - Problems 5, 7, 8
- Consider solving Example 1.3 with a table starting at  $a=1$ ,  $b=1$ ,  $c=1$ . How long would the table be if you step through the choices? How many entries would you check if you bisect the choices? (Hints:
  1. Don't make the lists, but rather count the steps in the method for making the list.
  2. Or just go ahead and use a program or spreadsheet to build the lists.)

Note that you *may* email homework. However, I don't use Microsoft<sup>TM</sup> products (*e.g.* Word), and software packages are notoriously finicky about translating mathematics.

If you're typing it (which I advise just for practice in whatever tools you use), you likely want to turn in a printout. If you do want to email your submission, please produce a PDF or PostScript document.