Math 202 notes

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8 September, 2008

Contents

Notes also available as PDF.

1 Review

set An *unordered* collection of *unique* elements.

- Curly braces: $\{A, B, C\}$ is a set of three elements, A, B, and C.
- Can be *implicit* or in *set builder notation*: $\{x \mid x \text{ is an integer}, x > 0, x < 3\}$ is the same set as $\{1, 2\}$.
- Order does not matter, repeated elements do not matter.

element Any item in a set, even other sets. (Also entry, member, item, *etc.*)

empty set Or null set. Denoted by $\emptyset$ rather than $\{\}$.

- This is a *set* on its own.
- $\{\emptyset\}$ is the set of the empty set, which is not empty.

singleton A set with only one element.

2 Relations and Venn diagrams

(Someday I will include Venn diagrams for these in the notes.)

element of The expression $x \in A$ states that x is an element of A. If $x \notin A$, then x is *not* an element of A.

- $4 \in \{2, 4, 6\}$, and $4 \notin \{x \,|\, x \text{ is an odd integer }\}$.
- There is no x such that $x \in \emptyset$, so $\{x \,|\, x \in \emptyset\}$ is a long way of writing $\emptyset$.

subset If all entries of set A also are in set B, A is a subset of B.

superset The reverse of subset. If all entries of set B also are in set A, then A is a superset of B.

proper subset If all entries of set A also are in set B, but some entries of B are *not* in A, then A is a *proper* subset of B.

- $\{2, 3\}$ is a proper subset of $\{1, 2, 3, 4\}$.

equality Set A equals set B when A is a subset of B and B is a subset of A.

- Order does not matter. $\{1, 2, 3\} = \{3, 2, 1\}$.

The symbols for these relations are subject to a little disagreement.

- Many basic textbooks write the subset relation as $\subseteq$, so $A \subseteq B$ when A is a subset of B. The same textbooks reserve $\subset$ for the *proper* subset. Supersets are $\supset$.
- This keeps a superficial similarity to the numerical relations $\leq$ and $<$. In the former the compared quantities may be equal, while in the latter they must be different.
- Most mathematicians now use $\subset$ for any subset. If a property requires a “proper subset”, it often is worth noting specifically. And the only non-“proper subset” of a set is the set itself.
- Extra relations are given for emphasis, *e.g.* $\subsetneq$ or $\subsetneqq$ for proper subsets and $\subseteq$ or $\subseteqq$ to emphasize the possibility of equality.
- Often a proper subset is written out: $A \subset B$ and $A \neq B$.
- **I’ll never remember to stick with the textbook’s notation. My use of $\subset$ is for subsets and not proper subsets.**

3 Translating relations into (and from) English

From English:

- The train has a caboose.
  - It's reasonable to think of a train as a set of cars (they can be reordered).
  - The cars are the members.
  - Hence, caboose $\in$ train
- The VI volleyball team consists of VI students.
  - VI volleyball team $\subset$ VI students
- There are no pink elephants.
  - pink elephants $= \emptyset$

To English:

- $x \in$ today's homework set.
  - x is a problem in today's homework set.
- Today's homework $\subset$ this week's homework.
  - Today's homework is a subset of this week's homework.

4 Consequences of the set relation definitions

Every set is a subset of itself. Expected.

If $A = B$, then every member of A is a member of B, and every member of B is a member of A. This is what we expect from equality, but we did not define set equality this way. Follow the rules:

- $A = B$ imples $A \subset B$ and $B \subset A$.
- Because $A \subset B$, every member of A is a member of B.
- Because $B \subset A$, every member of B is a member of A.

The empty set $\emptyset$ is a subset of all sets. Unexpected! This is a case of carrying the formal logic to its only consistent end.

- For some set A, $\emptyset \subset A$ if every member of $\emptyset$ is in A.
- But $\emptyset$ has no members.
- Thus all of $\emptyset$'s members also are in A.
- This is called a *vacuous* truth.

The alternatives would not be consistent, but proving that requires more machinery that we need.

5 Operations

union The *union* of two sets A and B, denoted by $A \cup B$, is the set consisting of all elements from A and B.

- $A \cup B = \{x \,|\, x \in A \text{ or } x \in B\}$.
- Remember repeated elements do not matter: $\{1,2\} \cup \{2,3\} = \{1,2,3\}$.

intersection The *intersection* of two sets A and B, denoted $A \cap B$, is the set consisting of all elements that are in *both* A and B.

- $A \cap B = \{x \,|\, x \in A \text{ and } x \in B\}$.
- $\{1,2\} \cap \{2,3\} = \{2\}$.
- $\{1,2\} \cap \{3,4\} = \{\} = \emptyset$.

set difference The *set difference* of two sets A and B, written $A \setminus B$, is the set of entries of A that are not entries of B.

- $A \setminus B = \{x \,|\, x \in A \text{ and } x \notin B\}$.
- Sometimes written as $A - B$, but that often becomes confusing.

If A and B share no entries, they are called *disjoint*. One surprising consequence is that every set A has a subset disjoint to the set A itself.

- No sets (not even $\emptyset$) can share elements with $\emptyset$ because $\emptyset$ has no elements.
- So all sets are disjoint with $\emptyset$.
- The empty set $\emptyset$ is a subset of all sets.
- So all sets are disjoint with at least one of their subsets!

Can any other subset be disjoint with its superset? *No.*

6 Homework

Groups are fine, turn in your own work. Homework is due in or before class on Mondays.

Write out (briefly) your approach to each problem.

- Problem set 2.1 (p83):
  - Problems 7, 8, 10, 20, 24

Note that you *may* email homework. However, I don't use Microsoft$^{\text{TM}}$ products (*e.g.* Word), and software packages are notoriously finicky about translating mathematics.

If you're typing it (which I advise just for practice in whatever tools you use), you likely want to turn in a printout. If you do want to email your submission, please produce a PDF or PostScript document.