

# Solutions for the fourth week's homework

## Math 202

Jason Riedy

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Also available as PDF.

**Note:** These are my approaches to these problems. There are many ways to tackle each.

### 1 Problem set 2.2

#### 1.1 Problem 1

- (a) June 13<sup>th</sup>: nominal, first: ordinal
- (b) eleventh: ordinal, second: ordinal, 6-iron: nominal, 160 yards: cardinal
- (c) 7 pin: nominal, sixth frame: ordinal, 9: cardinal

#### 1.2 Problem 2

- (a) There are five letters in the phrase,  $\{P,A,N,M,B\}$ , so we can associate  $1 \leftrightarrow P$ ,  $2 \leftrightarrow A$ , *etc.* The two sets are **equivalent**.
- (b) The second set has one more element than the first, so the sets are **not equivalent**.
- (c) We can associate  $o \leftrightarrow t$ ,  $n \leftrightarrow w$ , and  $e \leftrightarrow o$ , where the left quantities come from  $\{o, n, e\}$  and the right from  $\{t, w, o\}$ . The two sets are **equivalent**.
- (d)  $\{0\}$  has in element, and  $\emptyset$  has none, so the sets are **not equivalent**.

#### 1.3 Problem 6

- (a) We can use the function  $f(w) = w + 1$  to construct a bijection between  $W$  and  $N$ .
- (b) The function  $f(d) = d + 1$  creates a one-to-one mapping between  $D$  and  $E$ .

- (c) The function  $f(n) = 10^n$  creates a one-to-one mapping between  $N$  and  $\{10, 100, \dots\}$ .

### 1.4 Problem 13

The drawing will have

- $|B \cap C| - |A \cap B \cap C| = 5$  in the unlabeled portion of  $B \cap C$ ,
- $|B| - |A \cap B| - |B \cap C| + |A \cap B \cap C| = 28$  in the remaining unlabeled portion of  $B$ ,
- $|A \cap C| - |A \cap B \cap C| = 8$  in the unlabeled portion of  $A \cap C$ ,
- $|A| - |A \cap B| - |A \cap C| + |A \cap B \cap C| = 15$  in the remaining unlabeled portion of  $A$ ,
- $|C| - |B \cap C| - |A \cap C| + |A \cap B \cap C| = 10$  in the remaining unlabeled portion of  $C$ , and
- $|U| - (10 + 7 + 5 + 28 + 8 + 15 + 10) = 17$  in  $U$  but outside  $A$ ,  $B$ , and  $C$ .

### 1.5 Problem 21

- (a) The function  $f(a) = a$  is the one-to-one correspondence from  $A$  to itself.
- (b) If  $A \sim B$ , there is a function  $f(a) = b$  where each  $b \in B$  appears for exactly one  $a \in A$  and the function is defined for every  $b \in B$  and  $a \in A$ . The *inverse* function  $g(b) = a$  where  $b = f(a)$  shows  $B \sim A$ .
- (c) If  $A \sim B$  and  $B \sim C$ , then there are one-to-one functions  $f(a) = b$  and  $g(b) = c$  for each mapping. The function  $h(a) = g(f(a))$  then is a one-to-one mapping between  $A$  and  $C$ .

### 1.6 Problem 23

For (a) and (b), the table is Pascal's triangle, which we already have in the notes. For (c), we want the entry  $P_{6,3}$  in our earlier notation. There are 20 such ways.

### 1.7 Why answering problem 32 would be a bad idea.

Wow. Never, **ever** give someone a list of all your identifying numbers. Identity theft is a serious problem. While quite often all these numbers are available for a little work, at least make a criminal work for them.

## 2 Problem set 2.3

### 2.1 Problem 2

Substituting into  $|A \cup B| = |A| + |B| - |A \cap B|$ , we see that  $10 = 5 + 8 - |A \cap B|$ , or that  $|A \cap B| = 3$ .

### 2.2 Problem 5

I'm just going to show these as lines. Illustrating them on a "number line" is equivalent. A better diagram for the last would wrap portions of the result in parentheses to show how the line extended.

$$\begin{aligned} \text{(a)} \quad 3 + 5 &= \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet + \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 5 + 3 &= \\ &\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet + \bullet\text{---}\bullet\text{---}\bullet \\ &= \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 4 + 2 &= \\ &\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet + \bullet\text{---}\bullet \\ &= \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 0 + 6 &= \\ &\bullet + \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= 6 \end{aligned}$$

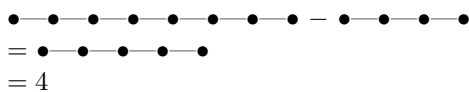
$$\begin{aligned} \text{(e)} \quad 3 + (5 + 7) &= \\ &\bullet\text{---}\bullet\text{---}\bullet + (\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet + \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet) \\ &= \bullet\text{---}\bullet\text{---}\bullet + \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad (3 + 5) + 7 &= \\ &(\bullet\text{---}\bullet\text{---}\bullet + \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet) + \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet + \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= \bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet \\ &= 15 \end{aligned}$$

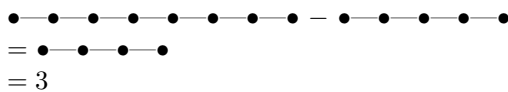
### 2.3 Problem 11

Again, I'm just going to show these as lines. Illustrating them on a "number line" is equivalent.

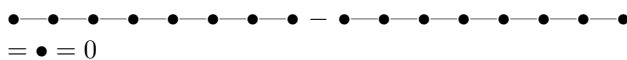
(a)  $7 - 3 =$



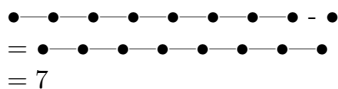
(b)  $7 - 4 =$



(c)  $7 - 7 =$



(d)  $7 - 0 =$



### 2.4 Problem 24

Draw with two shadings and show that the intersection is shaded twice.

## 3 Write $2 + 3$ using disjoint sets.

If we let  $2 \equiv \{a, b\}$  and  $3 \equiv \{c, d, e\}$ , then  $2 + 3 \equiv \{a, b\} \cup \{c, d, e\} = \{a, b, c, d, e\}$ .

## 4 Illustrate $2 + 3$ using Peano arithmetic.

We defined addition with

$$a + 0 = a, \text{ and}$$
$$a + S(b) = S(a + b).$$

Here,  $3 = S(2)$ , so

$$\begin{aligned}2 + 3 &= 2 + S(2) \\ &= S(2 + 2) \\ &= S(2 + S(1)) \\ &= S(S(2 + 1)) \\ &= S(S(2 + S(0))) \\ &= S(S(S(2 + 0))) \\ &= S(S(S(2))) \\ &= 5.\end{aligned}$$

## 5 Problem set 2.4

### 5.1 Problem 5

- (a) Any set that contains only 1 and some other number is **closed** because 1 is the multiplicative identity.
- (b) Any set that contains only 1 and some other number is **closed** because 1 is the multiplicative identity.
- (c)  $2 \cdot 4 = 8 \notin \{0, 2, 4\}$ , so this is **not closed**.
- (d) The product of even numbers always is even, so this is **closed**.
- (e) The product of odd numbers cannot be even, so they must be odd and this set is **closed**.
- (f)  $2^2 \cdot 2^3 = 2^5 \notin \{1, 2, 2^2, 2^3\}$ , so this set is **not closed**.
- (g) The product of powers of two is a power of two, so this set is **closed**.
- (h) Similarly, the product of powers of seven is a power of seven so this set is **closed**. Both of these are closed because  $\{0, 1, 2, \dots\}$  is closed under addition;  $a^i \cdot a^j = a^{i+j}$ , moving the property up to the superscript.

### 5.2 Problem 10

Each product is equal to the appropriate shaded area, and the sum is equal to the entire rectangle. The letters correspond directly to the positions.

*I had completely forgotten about the FOIL mnemonic. After long enough, it's just a part of what you do.*

### 5.3 Problem 26

The operation is **closed** because no new shapes are introduced in the operator's table of results.

The operation is **commutative** because the operation table is symmetric across the diagonal axis. Thus  $a \star b = b \star a$ .

Because  $\circ \star x = x$ , the  $\circ$  symbol is  $\star$ 's identity.

After the identity, there are only two symbols remaining. Thus the commutative property combined with the identity means that *this* operator must be **associative**.

## 6 Illustrate $2 \cdot 3$ using Peano arithmetic. You do not need to expand addition.

We defined multiplication with

$$\begin{aligned} a \cdot 0 &= 0, \text{ and} \\ a \cdot S(b) &= a + (a \cdot b). \end{aligned}$$

So

$$\begin{aligned} 2 \cdot 3 &= 2 \cdot S(2) \\ &= 2 + (2 \cdot 2) \\ &= 2 + (2 \cdot S(1)) \\ &= 2 + (2 + (2 \cdot 1)) \\ &= 2 + (2 + (2 \cdot S(0))) \\ &= 2 + (2 + (2 + 2 \cdot 0)) \\ &= 2 + 2 + 2 = 6. \end{aligned}$$

## 7 Illustrate $(1 \cdot 2) \cdot 3 = 1 \cdot (2 \cdot 3)$ using a volume of size six.

Draw two  $1 \times 2 \times 3$  boxes made of  $1 \times 1 \times 1$  boxes. To illustrate  $(1 \cdot 2) \cdot 3$ , show that the  $1 \times 2$  portion is stacked 3 times. To illustrate  $1 \cdot (2 \cdot 3)$ , show that the  $2 \times 3$  portion is stacked across 1 time.