

Solutions for the ninth week's homework

Math 202

Jason Riedy

20 October, 2008

Also available as PDF.

Note: These are my approaches to these problems. There are many ways to tackle each.

1 Diophantine equations

See the previous week's solutions.

2 Problem set 6.1

Problem 2 I'm not going to draw this, but it should be fairly straight-forward.
This was more an exercise in something handy for elementary classes.

Problem 6 • $\frac{20}{60} = \frac{1}{3}$

- $\frac{30}{60} = \frac{1}{2}$
- $\frac{5}{7}$
- $\frac{25}{100} = \frac{1}{4}$
- $\frac{25}{100} = \frac{1}{4}$
- $\frac{3}{12} = \frac{1}{4}$
- $\frac{2}{3}$
- $\frac{3}{4}$ (A quart is made of *quarters*... There are four cups in a *quart*.
Cooking is a source of bizarre but traditional units.)

Problem 11 • $\frac{4}{5} = \frac{6 \cdot 4}{6 \cdot 5} = \frac{24}{30}$

- $\frac{6}{9} = \frac{3 \cdot 2}{3 \cdot 3} = \frac{2}{3}$
- $\frac{-7}{25} = \frac{20 \cdot -7}{20 \cdot 25} = \frac{-140}{500}$
- $\frac{18}{3} = \frac{3 \cdot 6}{3 \cdot 1} = \frac{6}{1} = \frac{-1 \cdot 6}{-1 \cdot 1} = \frac{-6}{-1} = 6$

Problem 12 • $\frac{18}{42} = \frac{6 \cdot 3}{6 \cdot 7} = \frac{3}{7}$.

- Here $(18, 49) = 1$ and $(5, 14) = 1$, so both are in lowest common terms. That form is unique, and these fractions differ, so these **fractions cannot be equal**. Following the text's method, you want to compare $\frac{2 \cdot 18}{2 \cdot 49} = \frac{36}{98}$ and $\frac{7 \cdot 5}{7 \cdot 14} = \frac{35}{98}$, because $\text{lcm}(49, 14) = 98$.
- $\frac{9}{25} = \frac{20 \cdot 9}{20 \cdot 25} = \frac{180}{500} \neq \frac{140}{500}$.
- $\frac{24}{144} = \frac{2}{12} = \frac{1}{6}$, $\frac{32}{96} = \frac{1}{3} = \frac{2}{6}$. **These are not equal.**

Problem 32 Yes, this is a general property. But you need to provide some examples.

3 Problem set 6.2

Problem 6 • $\frac{5}{7}$

- $\frac{10}{5} = 2$
- $\frac{20}{24} = \frac{5}{6}$
- $\frac{56}{65}$
- $\frac{76}{60} = \frac{19}{15}$
- $\frac{100}{200} = \frac{1}{2}$
- $\frac{-31}{100}$
- $\frac{321}{450} = \frac{107}{150}$

Problem 18 • 1

- $\frac{1}{4}$
- 1

Problem 13 • $3 \cdot \frac{5}{2} = \frac{15}{2}$

- $\frac{2}{3} \cdot \frac{3}{2} = 1$
- $\frac{3}{4} \cdot 2 = \frac{3}{2}$

Problem 25 For each, you proceed by solving the row, column, or diagonal that has only one open spot. Repeating suffices to fill the squares.

- | | | |
|---------------|----------------|----------------|
| $\frac{1}{2}$ | $\frac{1}{12}$ | $\frac{5}{12}$ |
| $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{5}{12}$ |
| $\frac{1}{4}$ | $\frac{7}{12}$ | $\frac{1}{6}$ |
- | | | |
|----------------|----------------|----------------|
| $\frac{8}{15}$ | $\frac{1}{5}$ | $\frac{4}{15}$ |
| $\frac{1}{15}$ | $\frac{2}{5}$ | $\frac{3}{5}$ |
| $\frac{2}{5}$ | $\frac{7}{15}$ | $\frac{2}{15}$ |

4 Problem set 6.3

Problem 2 • Reassociate to add the first two terms with the common denominator of 5 first.

- Commute the terms in the parenthesis and reassociate to add terms with common denominator of 4 first.
- Use the distributive property to pull out the $\frac{2}{3}$, then add the eighths.
- Commute and reassociate to multiply $\frac{3}{4} \cdot \frac{4}{3} = 1$ first.

Problem 8 • $\frac{2}{3} \cdot \frac{4}{7} + \frac{2}{3} \cdot \frac{3}{7} = \frac{2}{3} \cdot (\frac{4}{7} + \frac{3}{7}) = \frac{2}{3} \cdot 1 = \frac{2}{3}$

- $\frac{4}{5} \cdot \frac{2}{3} - \frac{3}{10} \cdot \frac{2}{3} = (\frac{4}{5} - \frac{3}{10}) \cdot \frac{2}{3} = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$
- $\frac{4}{7} \cdot \frac{3}{2} - \frac{4}{7} \cdot \frac{6}{4} = \frac{4}{7} \cdot (\frac{3}{2} - \frac{6}{4}) = \frac{4}{7} \cdot 0 = 0$
- $(\frac{4}{7} \cdot \frac{2}{5}) / \frac{2}{7} = \frac{4 \cdot 2}{7 \cdot 5} \cdot \frac{7}{2} = \frac{4}{5}$

Problem 9 • adding fractions with a common denominator

- multiplying fractions
- distributing multiplication over addition
- adding fractions with a common denominator
- multiplying fractions

Problem 12 • First subtraction of 32, then multiplication by the inverse of $\frac{9}{5}$.

- | | | | | | | | |
|------------|-----|-----|----|----|----|-----|-----|
| Celsius | -40 | -20 | 0 | 10 | 20 | 45 | 100 |
| Fahrenheit | -40 | -13 | 32 | 50 | 68 | 104 | 212 |

- As seen in the table above, both agree at -40 . If $-F = C$, then $-F = \frac{5}{9}(F - 32)$, $-\frac{9}{5}F = F - 32$, $32 = \frac{14}{5}F$, $F = \frac{80}{7} \approx 11$.